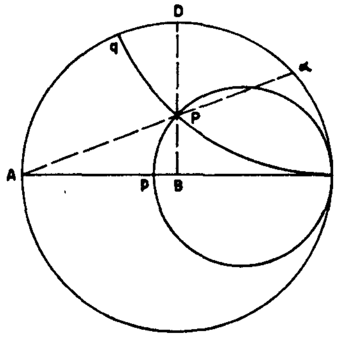

Fig. 1. Smith chart.

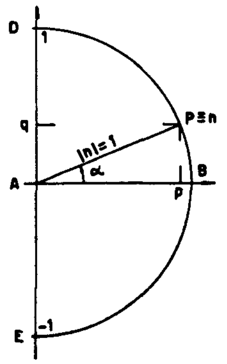

Fig. 2. n plane.

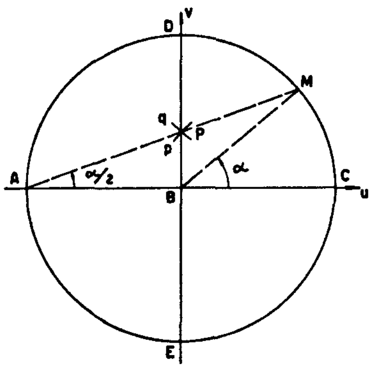

Fig. 3. w plane.

The proof is simple. The Smith chart originates from the conformal mapping of the plane of the normalized complex variable $n=p+jq$ over the plane of the complex variable $w=u+jv$, through the bilinear transformation $w=(n-1)/(n+1)$.

On plane n, the locus of points with unit modulus $|n|=1$, is the circle BDE (Fig. 2).

A point P on this circle can be represented either by its polar or rectangular coordinates as

$$P = n = 1 \cdot e^{j\alpha} = \cos\alpha + j\sin\alpha.$$

The image of the DBE circle on plane w is the segment DBE (Fig. 3).

Coordinates of P on the plane w are obtained directly from the transformation

$$w = u + jv = \frac{n-1}{n+1} = \frac{e^{j\alpha}-1}{e^{j\alpha}+1} = 0 + j\tan\frac{\alpha}{2}\cdot$$

Therefore, $\overline{PB}=\tan\alpha/2$, and since $\overline{AB}=1$, it follows that angle $(PAB)=\alpha/2$, and consequently angle $(MBC)=\alpha$.

This proves that segments AM and BD cross at point P corresponding to $p=\cos\alpha$, $q=\sin\alpha$.

References

[1] P. H. Smith, *Electronic Applications of the Smith Chart*. New York: McGraw-Hill, 1969, p. 166.

Comments on "A Quick Accurate Method to Measure the Dielectric Constant of Microwave Integrated-Circuit Substrates"

P. H. LADBROOKE, M. H. N. POTOK, AND E. H. ENGLAND

If, in the above short paper,[1] the data for open-edged resonators are plotted to a base of $\sqrt{m^2+n^2}$, where (n, m) characterizes the cavity mode, an excellent comparison with [1, Fig. 4(b)] is found. Such a plot is given in Fig. 1. It can therefore be asked whether Howell's frequency errors are of reactive origin, not resistive as he suggests. His results do not properly support the notion of resonant frequency change due to radiation loss, since there is no apparent correlation between the mode Q's and the frequency (or dielectric constant) errors for open sidewalls in Table I.

His results for closed-edge substrates are also consistent with reactive perturbation of the cavity. By positioning the apertures at the very corners, Howell seems to have achieved coupling to $\boldsymbol{H}$, without disturbing the electric field, in a region where the relative strength of $\boldsymbol{H}$ is mode independent, being essentially the boundary value. Hence, there should be an error due to magnetic perturbation of the same magnitude for all modes. Such a mode independence is shown by Howell's Table I[1]; a reasonable estimate for the associated error in ϵ would be +1.5 percent for closed sidewalls. The exact magnitude of this error depends upon the ratio of substrate thickness to coax conductor spacing, dielectric constant (i.e., how tightly the fields are bound to the dielectric), the coupling-aperture dimensions, and the detector sensitivity.

We have briefly examined a quartz slice of dimensions $75\times75\times2.5$ mm, prepared in the manner proposed by Howell. The results in Table I were obtained for the lowest few modes, to be compared with $\epsilon=3.85$ found by the methods in [1]. In this case, therefore, with approximately 2.5 mm bared at two diagonally opposite corners, the dominant effect was one of increased electric energy storage due to $\boldsymbol{E}$ no longer being zero at these corners. In view of the fact that in [1], for open sidewalls at least, the error was virtually identical for samples of Al_2O_3 and SiO_2 (being approximately five times thicker), it would be of interest to know the dimensions of Howell's corner apertures in his Al_2O_3 cavities such that his electric field was unaffected, whereas in our quartz this was no longer true.

Finally, if the central idea of our short paper is correct (namely that for this type of resonator, the frequency errors are dominated by reactive as opposed to resistive effects), then it should be possible to choose an optimum coupling point. Provided that perturbation of the electric field can be avoided, the best excitation for closed sidewalls probably is exactly that used by Howell; what recommends it is the error invariance among modes, so that only one or two spot measurements need be made. We have quickly tried this method with Al_2O_3 and N-type connectors and find the signal more difficult to detect than in the other techniques we have examined, due to exterior transmission paths as noted by Howell.

References

[1] P. H. Ladbrooke, M. H. N. Potok, and E. H. England, "Coupling errors in cavity resonance measurements on MIC dielectrics," this issue, pp. 560–562.

Manuscript received February 23, 1973.

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[1] J. Q. Howell, *IEEE Trans. Microwave Theory Tech.* (Short Paper), vol. MTT-21, pp. 142–143, Mar. 1973.

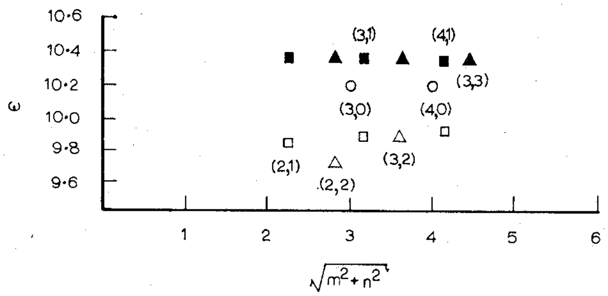

Fig. 1.

TABLE I

Frequency	Mode	Permittivity
2.22	(2,1)	3.94
2.86	(2,2)	3.82
3.20	(3,1)	3.82
3.64	(3,2)	3.82

Reply[2] *by J. Q. Howell*[3]

The purpose of my short paper[1] was to present an alternate method of measuring the dielectric constant of MIC substrates to that suggested by Napoli and Hughes [1]. My new scheme appeared to be more accurate and yet as easily implemented. I did not intend to imply that the error in the Napoli–Hughes technique was resistive in origin, but only that the measured resonant frequencies were found to be affected by the strength of the coupling and by radiation losses. This could be demonstrated by observing the shift in resonant frequencies when changing the coupling coefficient or when moving an object in the vicinity of the substrate. Ladbrooke *et al.* [2] did not mention error due to the fringing fields along the open edge of the substrate and they implied that the dominant error was due to the perturbation of the fields in the vicinity of the feedpoint. Since the closed sidewall technique more closely approximates the theoretical model, I submit that this scheme is the more accurate of the two. At any rate, the solution to the question of whether the resistive loss or the field perturbation causes the largest error in the resonant frequency will depend on the properties of the sample and equipment being used. Either extensive theoretical analysis or comparison to some other accurate measuring scheme will be required to properly resolve this.

[2] Manuscript received March 28, 1973.
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[1] L. S. Napoli and J. J. Hughes, "A simple technique for the accurate determination of the microwave dielectric constant for microwave integrated circuit substrates," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 664–665, July 1971.
[2] P. H. Ladbrooke, M. H. N. Potok, and E. H. England, "Coupling errors in cavity resonance measurements on MIC dielectrics," this issue, pp. 560–562.

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